remote-sensing-ms-intro

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Appropriate hypothesis tests of time trends in spatial data must account for spatial and temporal autocorrelation. The new method developed by Ives et al (unpublished) does so in an elegegant and efficient manor that traditional methods do not. However, as described, these methods can only be used to predict the effects of temporally constant factors. Here, I propose to extend the methods of Ives et al by allowing for the use of spatio-temporally autocorrelated predictors. I will test the extended methods by applying them to data simulated with the expected underlying structure. I will then apply the method to a real world data set to test the hypothesis that temperature is driving changes in phenology in certain land classes.

A spatio-temporally autocorrelated variable of interest follows the form:

where:

* is the value of the variable at pixel at time
* is the model intercept at pixel
* is the effect of the pixel’s value at the previous time point on the current value of
* is the effect of time on
* and is the random correlated error term:
  + is the correlation structure of where
  + is the spatial correlation matrix of ,
  + is a nugget that absorbs additional random variation among the s, and
  + is an identity matrix

And a predictor variable of interest follows the very similar form but with different parameter names for the sake of clarity:

## GLS

The GLS model using site-specific and non-temporally variable predictors (i.e. land cover classification) is performed by regressing the temporally collapsed response against them:

where

* is an length vector of temporally collapsed pixel-level time trend estimates from formula (1) obtained via CLS,
* is an design matrix of fixed site-level predictors (possibly including an intercept),
* is a length vector of effects for each predictor, and
* is the random error term whose covariance matrix is equivalent to which holds the spatial correlation and nugget for .

in this model, the best-linear unbiased estimators are obtained via the formula

and the expected variance of is

However, if the predictor is not constant over time, but is instead spatially and temporally variable (and autocorrelated), it is necessary to modify the process:

where has been replaced with which represents an design matrix of temporally collapsed predictors (i.e. CLS estimate estimate of from formula (2) ). Because the assumptions of GLS only involve the correlation pattern of the response, it is not necessary to account for the spatial component of directly. The regression should be able to detect when the variance structure of differs from the provided structure of . If additional variance is explained by , it should produce a significant .

To test this, I will apply the above method to simulated responses generated with a base-line spatio-temporal autocorrelation and the effect of a spatio-temporally autocorrelated predictor variable: .

## Variance Partitioning

The covariance structure of is . So, we should theoretically be able to recover the covariance structure of by estimating the differences between and (i.e ). This would give us the variation in our response that is not explained by variation in out spatio-temporally structured predictor . If the spatial correlations for and are equivalent (i.e. equal range of spatial autocorrelation; ), then the equation can be reduced to

where the nugget for is .

To test if is driving changes in , we would ask if variation in (including spatial variation) explains significant variation in . In other words, we would test against the null hypothesis that is a very small (0) proportion of (i.e. )

## Questions

With this method for predicting spatio-temporally correlated responses from spatio-temporally correlated predictors, we can appropriate test hypotheses about the effects of climate on ecological responses. For example, we can test the hypothesis that increasing temperature is associated with earlier onset of photosynthesis in particular land classes.